which can be measured directly.[9] This information includes the longitudinal spacing a, downstream distance na, where the vortex street ceases to be discernible, and the ambient wind speed u_o. Substitution for n in Eq. (3) yields an estimate of β. The corresponding estimate of eddy viscosity v is then made by adopting this value of β and N, determined from $N = u_e/a$, in the relation

$$v = \beta u_o^2/N \tag{4}$$

The results are contained in Table 1 which also lists corresponding estimates of the Reynolds number.

Conclusions

Evidence is provided in support of the Lin parameter being an inherent property of the vortex street. For the atmospheric vortex streets discussed here, β and v vary within the respective range of values given by

$$1.2 \times 10^{-3} < \beta < 2.0 \times 10^{-3}$$

and

$$2.2 \times 10^3 \text{ m}^2\text{sec}^{-1} < v < 4.4 \times 10^3 \text{ m}^2\text{sec}^{-1}$$

These values for the Gemini VI data are higher than those of Zimmerman.[4] Constancy of β and v throughout the wake region is implicit in these calculations. The Reynolds number for these island-generated vortex streets has values within the range

$$98 < Re < 183$$

the smaller values corresponding to the Madeira island.

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Accuracy of Complex Finite Elements

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Introduction

A NECESSARY condition for assuring the convergence of the finite element (Rayleigh-Ritz-Galerkin) method is that the trial function be complete in the energy.[1] This means that with a sufficiently fine mesh of finite elements the approximate energy can be made as close to the exact energy at the solution as desired. In problems of the $2m$th order (for harmonic problems $m = 1$, for biharmonic problems $m = 2$) the energy expression includes derivatives up to the mth order. The accuracy obtainable with the finite element trial function depends on how closely this function can approximate the exact solution up to its mth derivatives. Indeed, in proving the convergence of the finite element method, Synge[2,3] showed that any smooth function can be approximated by linear segments (finite elements) of size h up to $0(h^2)$. The first derivatives of this smooth function can be approximated by the linear elements up to $0(h)$. In second-order problems (heat transfer, potential flow, elasticity, etc.) the energy expression includes squares of the first derivatives. Therefore, this type of element permits the energy to be approximated up to $0(h^2)$. Since for obtaining the finite element approximate solution the energy is minimized, the actual error in the energy is at most $0(h^2)$. Evidently, using higher order interpolation schemes inside the element permits the attainment of a better approximation with the same number of elements and hence a higher rate of convergence. In this manner MacLay[4] (see also the thorough discussion in Ref. 5) established with the aid of the Taylor theorem that if the interpolation (mode, shape) functions inside the element include a complete polynomial of the pth degree, then the rate of convergence in the energy norm (square root of the energy) is $0(h^{p+1-m})$.

It is the purpose of this Note to show that the ability of the element to generate polynomials of a certain degree depends on the geometry of the element. In certain instances, as with highly curved elements, the amount of distortion in the element determines the degree of the complete polynomials in the interpolation functions. In the extreme, the element may become of the first order (as with a linear interpolation of variables inside it) independently of the number of variables associated with it. This then results in a loss of accuracy and efficiency.

Role of Polynomials

Consider a three-nodal-point one-dimensional element with nodes at $x = -h$, $x = 0$ and $x = h$. The displacement u inside it is interpolated by

$$u = u_1 \phi_1 + u_2 \phi_2 + u_3 \phi_3 \tag{1}$$

where u_i is the value of u at the ith nodal point and

$$\phi_1 = \xi(\xi-1)/2, \quad \phi_2 = 1-\xi^2, \quad \phi_3 = \xi(\xi+1)/2 \qquad \xi = x/h \tag{2}$$

Let u_i coincide with the exact displacement at the ith nodal point, and let $\tilde{u}$ be the displacement interpolated between the three nodal points. Expanding u in a Taylor series around $x = 0$ results in

$$\begin{aligned} u_1 &= u_2 - u_2' + (1/2!)u_2'' - (1/3!)u_2''' + \ldots \\ u_3 &= u_2 + u_2' + (1/2!)u_2'' + (1/3!)u_2''' + \ldots \end{aligned} \tag{3}$$

where $(\ )' = d/d\xi$. Introducing Eq. (3) into Eq. (1) results in

$$\tilde{u} = u_2(\phi_1+\phi_2+\phi_3) + u_2'(\phi_3-\phi_1) + (1/2!)u_2''(\phi_3+\phi_1) + \ldots \tag{4}$$

In Eq. (4) $\tilde{u}$ again denotes the interpolated displacement between the three nodal points and $u_2, u_2', u_2'', \ldots$ the exact values at the nodal point number 2 ($x = 0$). Since

$$\phi_1+\phi_2+\phi_3 = 1, \quad \phi_3-\phi_1 = \xi, \quad \phi_3+\phi_1 = \xi^2 \tag{5}$$

Eq. (4) becomes

$$\tilde{u} = u_2 + u_2'\xi + (1/2!)u_2''\xi^2 + (1/3!)u_2'''\xi + \ldots \tag{6}$$

The exact displacement u can be expanded around $x = 0$ in the form

$$u = u_2 + u_2'\xi + (1/2!)u_2''\xi^2 + (1/3!)u_2'''\xi^3 + \ldots \tag{7}$$

Subtracting Eq. (7) from Eq. (6) yields

$$\tilde{u} - u = (1/3!)u_2'''(\xi - \xi^3) + \ldots \tag{8}$$

Or, since $d^r u/d\xi^r = h^r d^r u/dx^r$, Eq. (8) gives

$$|\tilde{u} - u| \leq ch^3 \operatorname{Max} |d^3u/dx^3| \tag{9}$$

where c is a positive constant independent of h. In the more

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general case where the interpolation functions ϕ_i generate a polynomial of degree p

$$|\tilde{u}-u| \leq ch^{p+1}M_{p+1} \tag{10}$$

where M_{p+1} is a bound on the $(p+1)$th derivatives (with respect to x) of u.

In the same manner

$$|\partial^m(\tilde{u}-u)/\partial x^m| \leq ch^{p+1-m}M_{p+1} \tag{11}$$

is also obtained.

For problems of the 2mth order the energy includes derivatives of u up to the mth order, and therefore, with Eq. (10), the error in the energy norm (square root of the energy) $\|u-\tilde{u}\|_m$, becomes

$$\|\tilde{u}-u\|_m \leq c\,\text{Nes}^{-p-1+m}M_{p+1} \tag{12}$$

where Nes $= h^{-1}$ denotes the number of elements per side. Since the finite element solution is obtained by minimizing the energy (and hence the error in the energy norm), the actual error is at most that predicted by Eq. (12).

Simplex Elements

Linear, triangular, and tetrahedral elements are called simplex. They are characterized by the fact that in n dimensions they have $n+1$ vertices. To facilitate their analysis they are usually transformed from the Cartesian system $(0, x, y, z)$ into an analysis system $(0, \xi, \eta, \zeta)$ where the vertices are located at $(0,0,\ldots,0)$ $(1,0,\ldots,0)\ldots(0,0,\ldots,1)$. The transformation between the $(0xyz)$ system and the $(0, \xi, \eta, \zeta)$ system is linear as long as the sides of the elements are kept straight. Because of this linearity, if the interpolation functions inside the element include a complete polynomial of the pth degree in the ξ, η, ζ variables, it will also include a complete polynomial of the same degree in the x, y, z coordinates. It then results that the exponent of Nes in Eq. (12) will remain $-p-1+m$.

The next section deals with cases where, due to a nonlinear transformation of coordinates, inclusion of a complete polynominal of degree p in ξ, η, ζ may result in a lower order polynomial in x, y, z and, therefore, in a loss of the element's accuracy and efficiency.

Complex Elements

Elements which have more than $n+1$ vertices in n dimensions are termed complex. Otherwise simplex elements with curved sides are also included under this heading.

A common technique[6] in the analysis of these elements consists of transforming them from the $0xyz$ Cartesian coordinate system (the physical space) into the $0\xi\eta\zeta$ coordinate system (the analysis space), where they assume a regular form. Quadrilateral (curved) elements, for instance, are transformed into the cube (square) $0 \leq \xi \leq 1$, $0 \leq \eta \leq 1$ and $0 \leq \zeta \leq 1$. The interpolation functions inside the element consist of polynomials in $\xi\eta\xi$ such that the required compatibility between adjacent elements is maintained. In the isoparametric technique[6] the same functions are used for both the coordinate transformation and the interpolation of variable inside the element.

Let the transformation of coordination be written symbolically as $x = f(\xi)$, where x stands for the Cartesian and ξ for the Isoparameteric coordinates. If the variable u is interpolated inside the element by the same functions, then $u = f(\xi)$. Therefore, since $x = f(\xi)$ and $u = f(\xi)$, then $u = x$. This means that in the Isoparametric technique the interpolation functions always include the polynomial $1, x, y, z$, and p is at least 1 in Eq. (12). The condition that p be at least 1 assures the convergence of the method. However, p might be not higher than 1, resulting in a slowly convergent and hence poorly efficient element. In fact, if $x = x(\xi^q)$ and $u = u(\xi^p)$, then $u = u(x^{p/q})$. Also, if $p = q$ the element will be of the first order $(p = 1)$ independently of the number of degrees of freedom associated with it.

Several examples will now be considered. Let a mesh of finite elements be formed by intersecting curves. Let these (ξ, η) curves form a net of quadrilateral elements. If these are plate bending elements, continuity of displacements and slopes is achieved with the nodal variables (at the corners) w, w_ξ, w_η, $w_{\xi\eta}$ and with interpolation functions formed from the polynomials $1, \xi, \eta, \xi^2, \xi\eta, \eta^2, \xi^3, \xi^2\eta, \xi\eta^2, \eta^3, \xi^3\eta, \xi^2\eta^2, \xi\eta^3, \xi^3\eta^2, \xi^2\eta^3, \xi^3\eta^3$. This includes a complete polynomial of the 3rd degree, while the 4th degree is missing the terms ξ^4 and η^4. Making use of Taylor's theorem for obtaining the estimate for $|u-\tilde{u}|$ as given by Eq. (8), it is found that since the missing terms in the 4th degree polynomial are ξ^4 and η^4, the residual term in the expansion includes only $(\partial^4 u/\partial\xi^4)$ and $(\partial^4 u/\partial\eta^4)$, but no mixed derivatives. Assume now that the sides of the quadrilateral elements are straight. Then the transformation between the Cartesian system $(0xy)$ and the $(0\xi\eta)$ system can be given in the form

$$\begin{aligned} x &= (a_1\xi + a_2\eta + a_3\xi\eta)h \\ y &= (b_1\xi + b_2\eta + b_3\xi\eta)h \end{aligned} \tag{13}$$

where h is the diameter of the elements, and a_1, a_2, a_3, b_1, b_2 and b_3 are constants depending on the geometry of the element. The relation between derivatives with respect to ξ and those with respect to x are obtained by a successive application of

$$\begin{aligned} \partial/\partial\xi &= (\partial x/\partial\xi)\,\partial/\partial x + (\partial y/\partial\xi)\,\partial/\partial y \\ \partial/\partial\eta &= (\partial x/\partial\eta)\,\partial/\partial x + (\partial y/\partial\eta)\,\partial/\partial y \end{aligned} \tag{14}$$

The second derivative $\partial^2/\partial\xi^2$, for instance, is related to derivatives with respect to x by

$$\frac{\partial^2}{\partial\xi^2} = \left(\frac{\partial x}{\partial\xi}\right)^2 \frac{\partial^2}{\partial x^2} + 2\frac{\partial x}{\partial\xi}\frac{\partial y}{\partial\xi}\frac{\partial^2}{\partial x\partial y} + \left(\frac{\partial y}{\partial\xi}\right)^2 \frac{\partial^2}{\partial y^2} + \frac{\partial^2 x}{\partial\xi^2}\frac{\partial}{\partial x} + \frac{\partial^2 y}{\partial\xi^2}\frac{\partial}{\partial y} \tag{15}$$

It is important to note that $\partial^2/\partial\xi^2$ include lower order differentials of x, y. Now from Eq. (13) it results that $(\partial x/\partial\xi)^2 = 0(h^2)$ but that $(\partial^2 x/\partial\xi^2)$ could be only $0(h)$. However, since the coordinate transformation Eq. (13) includes only ξ, η, and $\xi\eta$, $\partial^2 x/\partial\xi^2 = 0$ and $\partial^2 y/\partial\eta^2 = 0$, and hence from Eq. (15) it results that

$$\frac{\partial^4}{\partial\xi^4} = \left(\frac{\partial x}{\partial\xi}\right)^4 \frac{\partial^4}{\partial x^4} + 4\left(\frac{\partial x}{\partial\xi}\right)^3\left(\frac{\partial y}{\partial\xi}\right) + \ldots + \left(\frac{\partial y}{\partial\xi}\right)^4 \frac{\partial^4}{\partial y^4} \tag{16}$$

The fact that $\partial^4/\partial\xi^4$ (and $\partial^4/\partial\eta^4$) does not include lower order derivatives of x assures that $\partial^4 u/\partial\xi^4 = 0(h^4)$. The residual term for the quadrilateral element will, therefore, be $0(h^4)$ and $|u-\tilde{u}| = 0(h^4)$, and according to Eq. (10) no loss of accuracy is registered. A loss of accuracy may arise with curved sides. If the element is curved the transformation of coordinates Eq. (13) will no longer be bilinear, and may include higher order powers of ξ and η. In such a case $(\partial^2 x/\partial\xi^2)$, $(\partial^3 x/\partial\xi^3)\ldots$ may not vanish, and, as indicated by Eq. (13), $\partial^4 u/\partial\xi^4$ may be only $0(h)$. Hence $|u-u| = 0(h^2)$. The error in the energy will then be $0(h)$, independently of the member of degrees of freedom associated with the element.

Another useful quadrilateral (curved) element formed by a net of curved lines is associated with the nodal variables (at the four corners) w, w_ξ, w_η, $w_{\xi\xi}$, $w_{\eta\eta}$, $w_{\xi\eta}$. Continuity of displacements and slopes is assured by constructing the interpolation functions from the polynomials $1, \xi, \eta, \xi^2, \xi\eta, \eta^2, \xi^3, \xi^2\eta, \xi\eta^2, \eta^3, \xi^4, \xi^3\eta, \xi^2\eta^2, \xi\eta^3, \eta^4, \xi^5, \xi^3\eta^2, \xi^2\eta^3, \eta^5, \xi^4\eta^2, \xi^3\eta^3, \xi^2\eta^4, \xi^5\eta^2, \xi^4\eta^3, \xi^3\eta^4, \xi^2\eta^5, \xi^5\eta^3, \xi^4\eta^4, \xi^3\eta^5$. This includes a complete polynomial of the 4th degree while that of the 5th degree is missing the terms $\xi^4\eta$ and $\eta\xi^4$. The residual expression of the Taylor expansion includes, therefore, the terms $(\partial^5 u/\partial\eta\,\partial\xi^4)$ and $(\partial^5 u/\partial\eta^4\,\partial\xi)$. In this case $\partial^5 u/\partial\eta\,\partial\xi^4$ and $\partial^5 u/\partial\eta^4\,\partial\xi$ include lower order differentials in x and y even for the case of elements with straight sides. For elements which deviate greatly from the parallelogram form [a_3 and b_3 in Eq. (13) are comparable in magnitude to a_1 a_2 b_1 and b_2] and the interpolation functions include a complete polynomial of only the 3rd degree in x and y instead of the 4th degree which are included in the case of a parallelogram element. Since in bending problems $m = 2$ $(2m = 4)$, the accuracy in the energy norm, according to Eq. (12), can be reduced in such elements from 0 (Nes^{-3}) to 0 (Nes^{-2}).

Consider now a quadrilateral (curved) element for plane elasticity having 9 nodal points, 3 on a side. Compatibility of displacements is assured if the interpolation functions include 1, ξ, η, ξ^2, $\xi\eta$, η^2, $\xi^2\eta^2$, $\xi\eta^2$, $\xi^2\eta^2$. Again, since the third degree polynomial is missing only ξ^3 and η^3, there will be no loss of accuracy in nonparallelogram elements with straight sides. When strongly curved, this (or any other isoparametric) element yields an error in the energy norm of only $0(\text{Nes}^{-1})$ as with linear interpolation of variables inside it.

The relative error of the rth eigenvalue in finite element eigenproblems is given by[7,8]

$$\delta\lambda_r/\lambda_r \leqq c\,\text{Nes}^{-2(p+1-m)}(\lambda_r/\lambda_1)^{(p+1-m)/m} \qquad (17)$$

and hence the same results for static problems carry over to dynamic problems. A more detailed discussion of this subject is to be found in Ref. 7.

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Turbulent Boundary Layers with Uniform Mass Transfer Including Low Reynolds Numbers

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Introduction

According to the experimental evidence, the turbulent boundary layer can be described as a composite layer consisting of inner and outer regions. In the inner region the mean velocity distribution may be described by the "law of the wall" as:

$$u^+ = \Phi_1(y^+) \qquad (1)$$

where $u^+ = u/u^*$, $y^+ = yu^*/\nu$, $u^* = (\tau_w/\rho)^{1/2}$. A popular eddy-viscosity expression that is used in this region is one based on Prandtl's mixing length theory

$$\varepsilon = l^2|\partial u/\partial y| \qquad (2)$$

or in dimensionless quantities

$$\varepsilon^+ = (l^+)^2|\partial u^+/\partial y^+| \qquad (3)$$

where $l = ky$, $l^+ = lu^*/\nu$, $\varepsilon^+ = \varepsilon/\nu$. In the case of a smooth wall, the inner region contains a laminar sublayer, adjacent to the wall, where the flow is primarily viscous. In this region the mean velocity increases linearly with distance from the wall and is given by

$$u^+ = y^+ \qquad (4)$$

Experimental evidence indicates that there is still another transitional region between the sublayer and the inner region where the flow is neither completely viscous nor completely turbulent. To predict the velocity distribution continuously down to the wall, Van Driest[1] proposed the following modification to Prandtl's mixing length theory:

$$l = ky[1-\exp(-y/A)] \qquad (5)$$

In terms of dimensionless quantities, Eq. (5) can be written as

$$l^+ = ky^+[1-\exp(-y^+/A^+)] \qquad (6)$$

where $A^+ = 26$, $k = 0.4$.

Although the aforementioned expression is very useful for calculating velocity and shear distributions in turbulent flows, it is primarily restricted to flows with no mass transfer and no pressure gradients. Cebeci[2] extended Van Driest's expression to turbulent flows with mass transfer and pressure gradients. According to Ref. 2, for a flat plate flow with mass transfer, the damping constant, A^{++}, given by

$$A^{++} = A^+\exp(-5.9\text{v}_w{}^+) \qquad (7)$$

should replace A^+ in Eq. (6). The constants $A^+ = 26$ and $k = 0.4$ were obtained empirically from experimental data at high Reynolds numbers. In Ref. 3, Cebeci repeated Van Driest's procedure for low Reynolds numbers and expressed these constants as functions of Reynolds number.

$$A^+ = -5.3885\times10^{-3}R_\theta + 29.966 + (31.291\times10^{-6}R_\theta{}^2 - 74.305\times10^{-3}R_\theta + 126.2)^{1/2} \qquad (8)$$

$$k = -5.1425\times10^{-5}R_\theta + 0.4485 + (2.851\times10^{-9}R_\theta{}^2 - 9.11\times10^{-6}R_\theta + 2.296\times10^{-2})^{1/2} \qquad (9)$$

where R_θ is the Reynolds number based on momentum thickness. The purpose of this Note is to investigate the accuracy of the proposed modifications to the mixing length in predicting the velocity distributions with uniform mass transfer for both low and high Reynolds numbers.

Analysis

If the normal stress terms are neglected, the incompressible turbulent boundary-layer equations for a two-dimensional flow can be written as

$$\text{Continuity:}\quad \partial u/\partial x + \partial v/\partial y = 0 \qquad (10)$$

$$\text{Momentum:}\quad u\,\partial u/\partial x + v\,\partial u/\partial y = -1/\rho(dp/dx) + (1/\rho)\,\partial\tau/\partial y \qquad (11)$$

where τ is the total shear stress, given by

$$\tau = \mu\,\partial u/\partial y - \rho\overline{u'v'} = \tau_l + \tau_t \qquad (12)$$

Introducing Boussinesq's eddy-viscosity concept to relate the Reynolds shear stress to the velocity distribution, namely, $-\rho\overline{u'v'} = \rho\varepsilon\,\partial u/\partial y$, Eq. (12) can be written as

$$\tau = \mu\,\partial u/\partial y + \rho\varepsilon\,\partial u/\partial y \qquad (13)$$

Now consider flat plate flow, assume local similarity, and neglect the $\partial u/\partial x$ term. Equations (10) and (11) can then be written as

$$dv/dy = 0 \qquad (14)$$

$$\rho v\,du/dy = d\tau/dy \qquad (15)$$

with the following boundary conditions:

$$y = 0,\quad u = 0,\quad v = 0 \ \text{ or } \ v_w\,(\text{mass transfer}),\quad \tau = \tau_w \qquad (16)$$

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